

Strategies for Helping Pre-Algebra Students Develop Symbol Sense in Grades 6-8

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TODOS

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What are we talking about?

NUMBER SENSE:

"[Number sense] is a comfort with what numbers represent, coming from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of numbers, such as fractions and decimals, are related to each other, and how they can best be used to describe a particular situation. Number sense is an attribute of all successful users of mathematics."

What are we talking about?

SYMBOL SENSE

Symbol sense is a comfort with what variables and expressions represent, coming from investigating their characteristics and using them in diverse situations. It involves an understanding of how different types of variables and expressions are related to each other, and how they can best be used to describe a particular situation. Symbol sense is an attribute of all successful users of mathematics."

Overlay

- How does this topic apply to ELL students?
 - Abstract thinking is essential for success in mathematics.
 - ELL students often face special challenges when transitioning to symbolic thinking in mathematics.
- What does the research say?
 - This does NOT have to be the case!

Two Essential Studies

- **Project Challenge**
 - Communication is necessary for building understanding
- **The Case of Railside School**
 - Students with diverse mathematics classes especially thrive with the use of words, symbols, and graphs in a “communicative classroom”.

Goals of Session

- Consider examples of ways to build understanding and facility with variables and variable expressions in our students
- Think together about helping students develop deep symbol sense
- Challenge each of you to employ these ideas into your thinking and teaching

Working with Variables and Variable Expressions

Variable - a number or set of numbers represented by a symbol

Variable Expression - a mathematical phrase containing one or more variables and may include operations

Working with Variables

Essential Understanding

Variables have many different meanings, depending on context and purpose

Mathematical Contexts

- Stand for a specific number
- State an identity, true for all numbers
- Stand for a range of numbers

ASIDE

Avoiding Confusion

When choosing variables, ... students should specify a unit...

Example: There are 3 feet in every yard.

When asked to convert feet to yards, students often write

$$3f = y$$

Instead, we want them to write

$f = \underline{\text{number}}$ of ft and $y = \underline{\text{number}}$ of yds

$$\text{so } 3y = f$$

Find Out What Your Students Know

1. $y = 3x + 4$	5. $P = 2l + 2w$	9. $3 + 2n = 8$
2. $40 = 5x$	6. $3x + 2y$	10. $a(b + c) = ab + ac$
3. $5x + 2x = 5x$	7. $a + b = b + a$	11. $5x - 2x$
4. $A = bh$	8. $3y - 5x = 17$	12. $d = rt$

- 1) Sort the cards in any way that makes sense to you.
- 2) Write a sentence or two that describes the symbol strings that you have grouped together.
- 3) Write another symbol string for each sorted group.

Sorting Symbol Strings

I. $5x - 2x$

$3x + 2y$

Incomplete Equations

No solutions

New: $4y + 3w$

II. $d = rt$

$y = 3x + 4$

$P = 2l + 2w$

$A = bh$

Functions and Formulas

Equations with two or
more variables

New: $A = \pi r^2$

Sorting Symbol Strings

III. $a + b = b + a$

$$a(b + c) = ab + ac$$

$$3x + 2x = 5x$$

Identities

Equations that are
always true

New: $2(6x) = 12x$

IV. $3y - 5x = 17$

$$40 = 5x$$

$$3 + 2n = 8$$

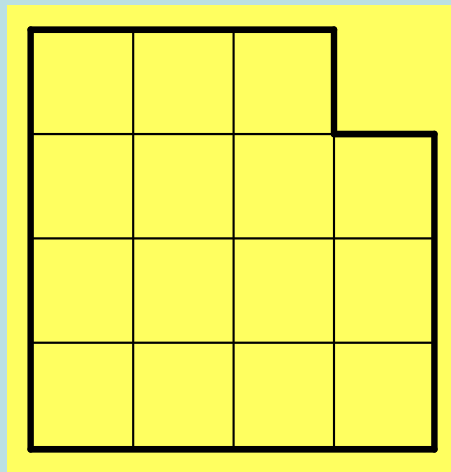
One-sided Equations

Equations with
variables on
only one side

New: $3x + 4 = 22$

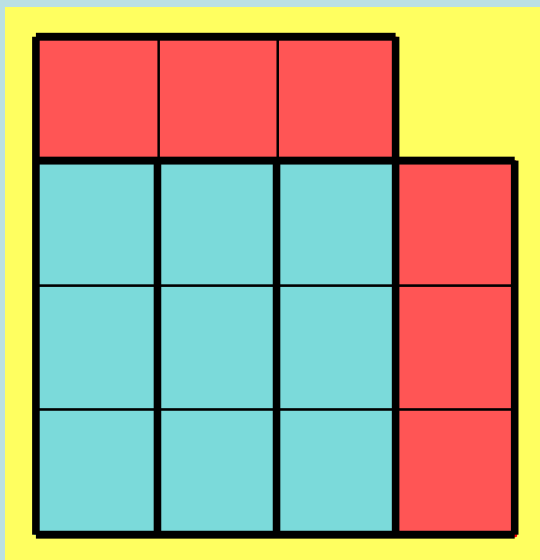
Bridge from Number Activities

Start with pictures, diagrams, and number activities.



Use shading to show one way this set of gym lockers with square openings might be divided into different groups. Write a matching numerical expression.

Example:



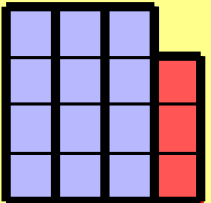
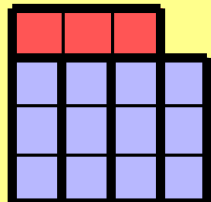
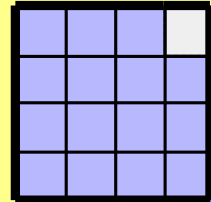
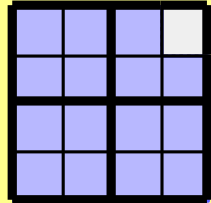
Suppose you shaded the lockers as shown at the left. You would have:

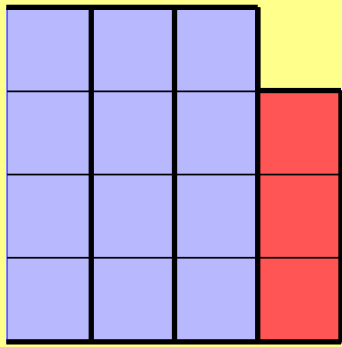
- 3 blue parts each 3 rows high by 1 column
- 2 red parts, one 1 row by 3 columns and the other, 3 rows by 1 column

You might write

$$3(3 \cdot 1) + (1 \cdot 3) + (3 \cdot 1)$$

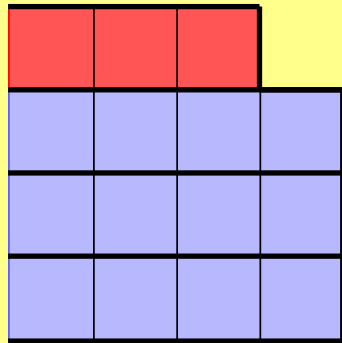
Other Interpretations

VIEW	WHAT I SEE
	$(4 \times 3) + (3 \times 1)$
	$3(1 \times 4) + (1 \times 3)$
	$(4 \times 4) - 1$
	$4(2 \times 2) - 1$

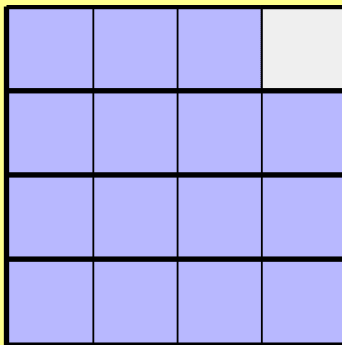


**4 x 4
Expression**

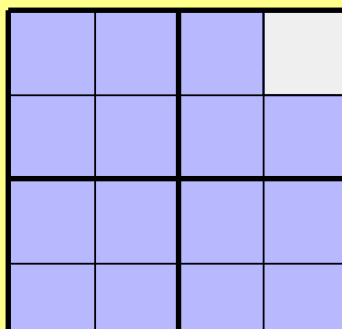
$$(4 \times 3) + (3 \times 1)$$



$$(3 \times 4) + (1 \times 3)$$



$$(4 \times 4) - 1$$



$$4(2 \times 2) - 1$$

**10x10
Expression**

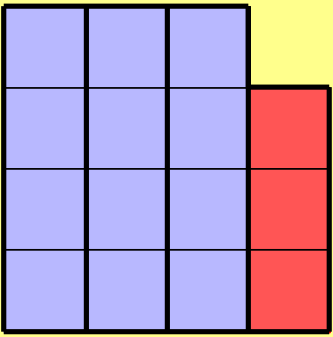
$$(10 \bullet 9) + (9 \bullet 1)$$

$$(9 \bullet 10) + (1 \bullet 9)$$

$$(10 \bullet 10) - (1)$$

$$4(5 \bullet 5) - 1$$

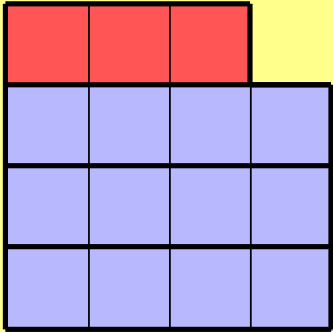
10 × 10
Expression



$$(10 \cdot 9) + (9 \cdot 1)$$

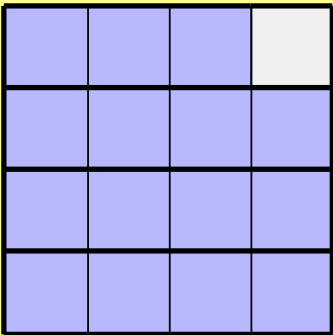
Symbolic
Expression

$$n(n-1) + (n-1)1$$



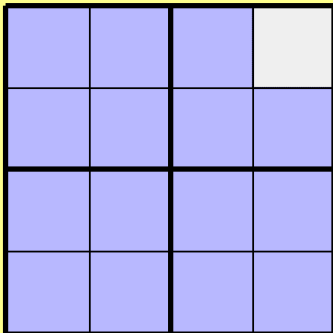
$$(10 \cdot 9) + (1 \cdot 9)$$

$$[n(n-1)] + 1(n-1)$$



$$(10 \cdot 10) - 1$$

$$(n \cdot n) - (1)$$



$$4(5 \cdot 5) - 1$$

$$4[(n/2)(n/2)] - 1$$

What is gained through bridging activities?

- Students transfer what they find in one numerical situation to an analogous numerical situation.
- Students transfer their findings about a numerical pattern to a general form, using variables
- Students begin to consider what they know about arithmetic to help them make sense of situations involving variables.

ASIDE
ASIDE

Doing and Undoing

If the general expression for a set of lockers is $x^2 + 1$, what does the set of lockers look like?

Variables on a Number Line

Partner Questions

- What do you know now?
(Do you agree? Why?)
- How do you know?
(Do you agree? Why?)
- Do you know the value of x ?
(Do you agree? Why?)

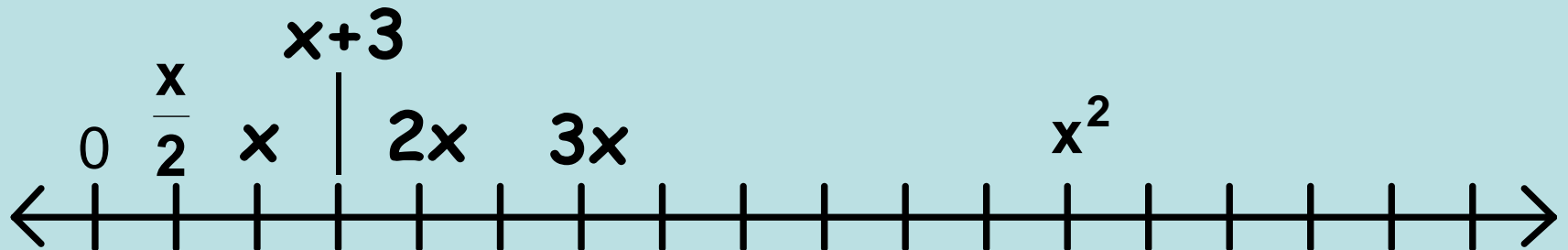
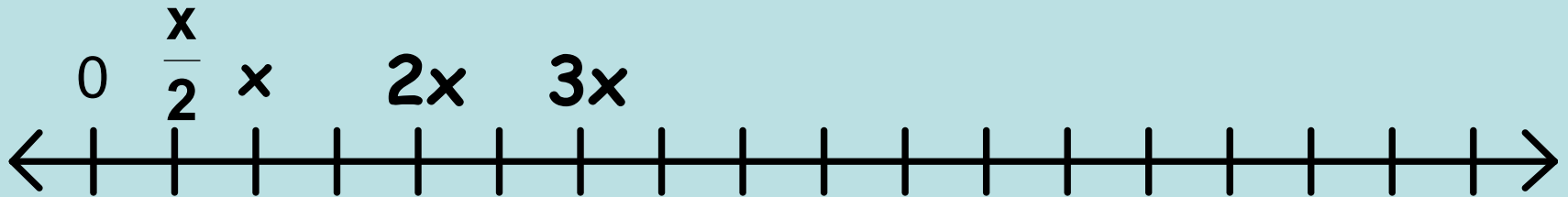
Variables on a Number Line

$0, x, \frac{x}{2}, 2x, 3x, x + 3, x^2, x - 4,$



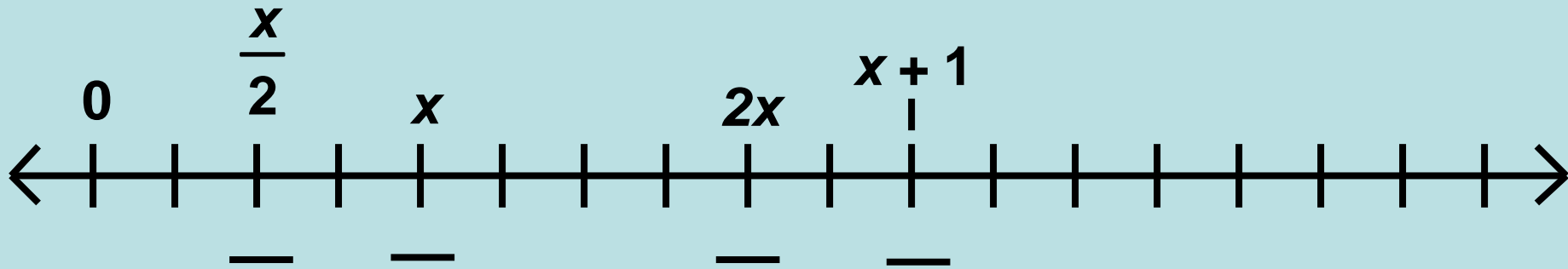
Variables on a Number Line

$x, \frac{x}{2}, 2x, 3x, x + 3, x^2, x - 4,$



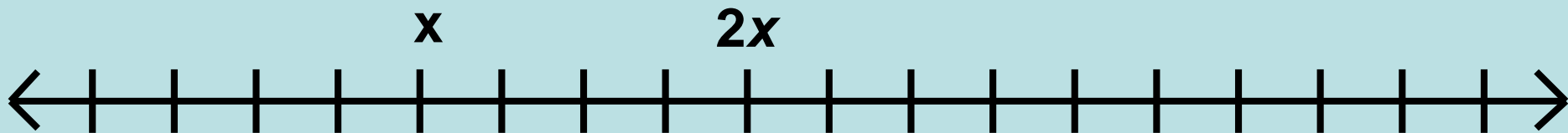
Follow-up Questions

1. Fill in all values for the variable expressions on the number line.



2. Add any expressions that you can *know* where they must go on the number line

$$x + 5, 3x, 0, \frac{x}{2}, x - 3$$

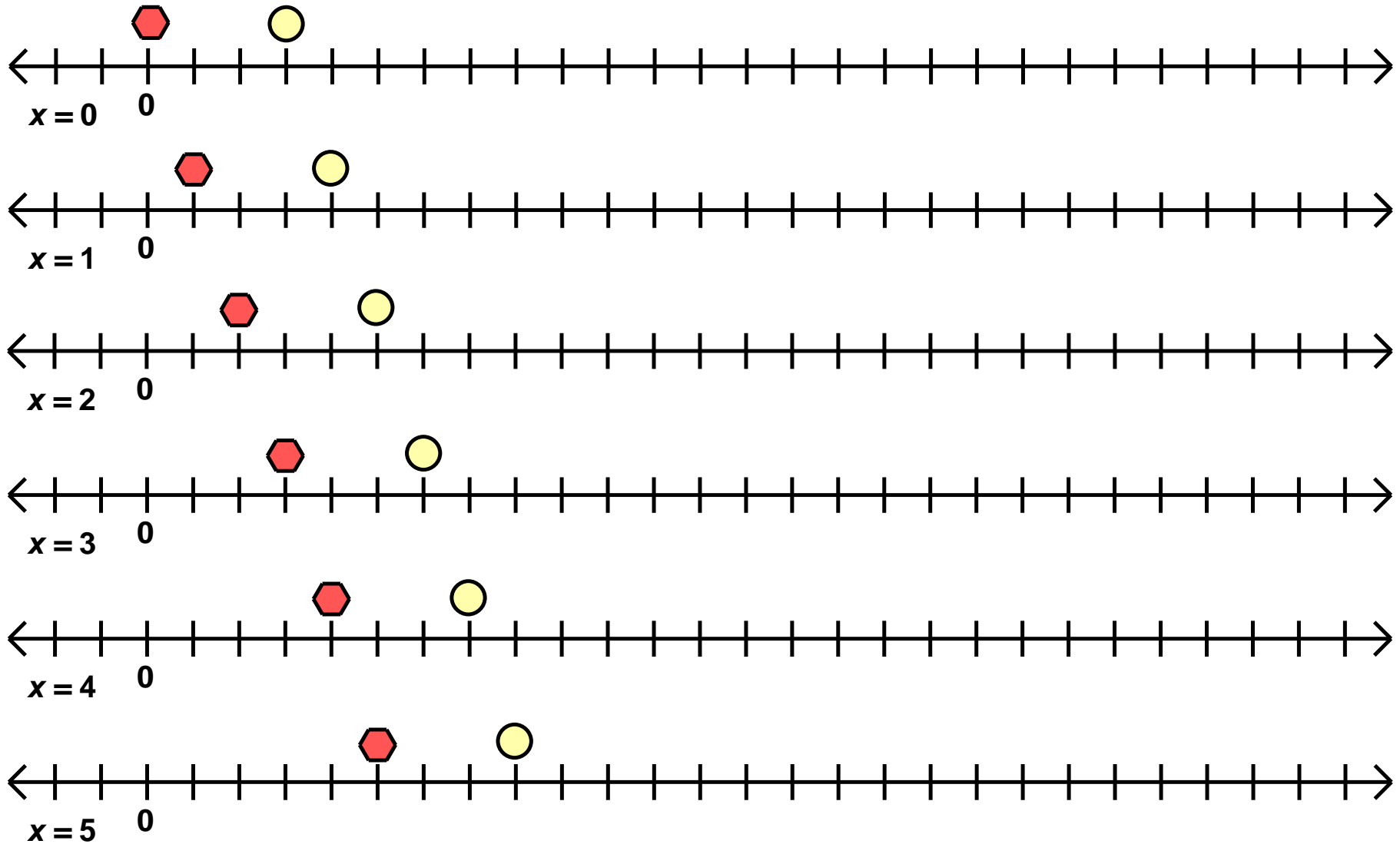


Benefits of Variables on a Scaled Number Line

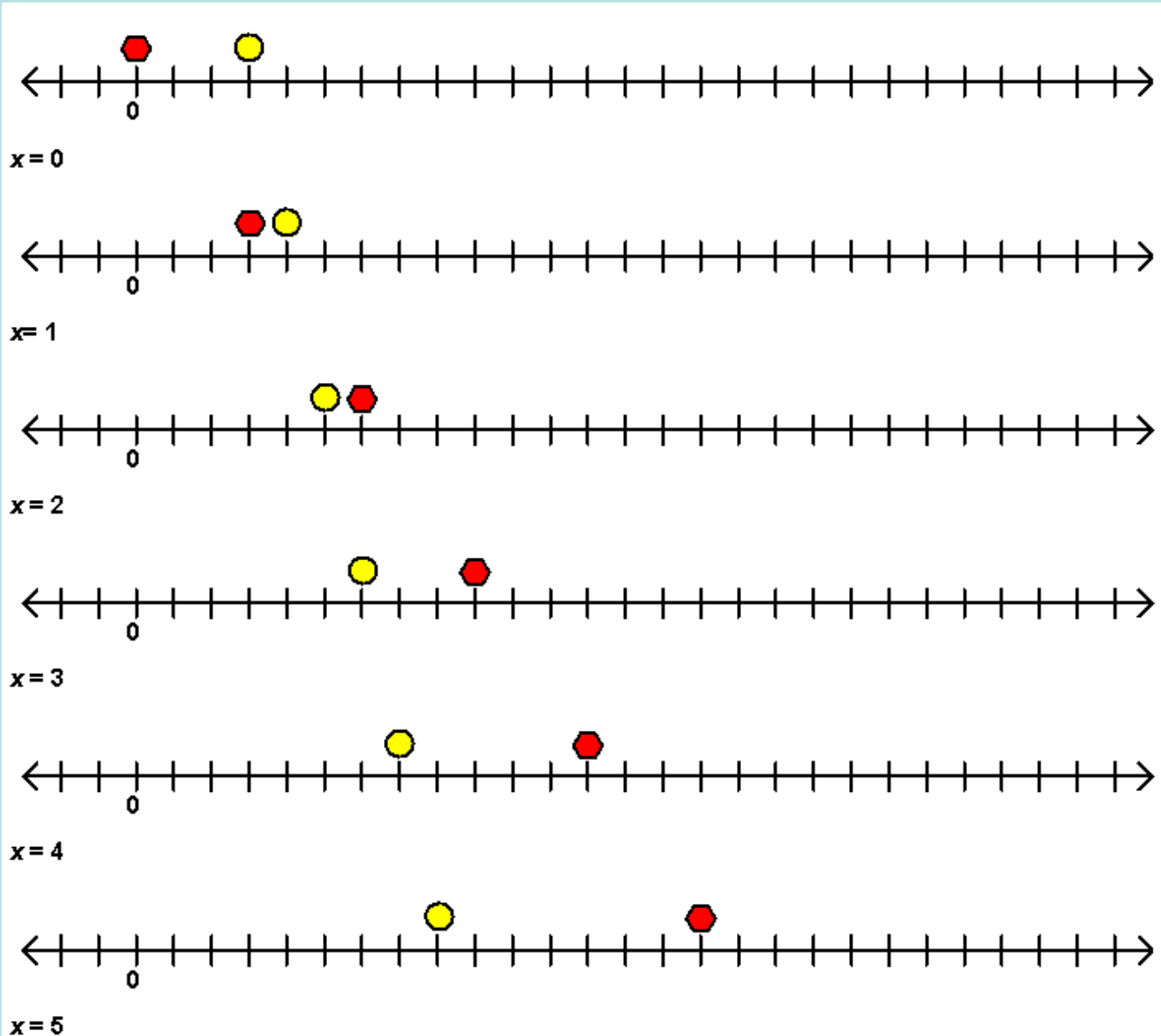
(scaled & one or more values for variable)

- Students re-examine sets of numbers and their behavior under operations.
- Students communicate their thinking: make mathematical conjectures, arguments for and against conjectures, and conclusions.
- Students develop more comfort with and understanding of symbols (more **symbol sense**).

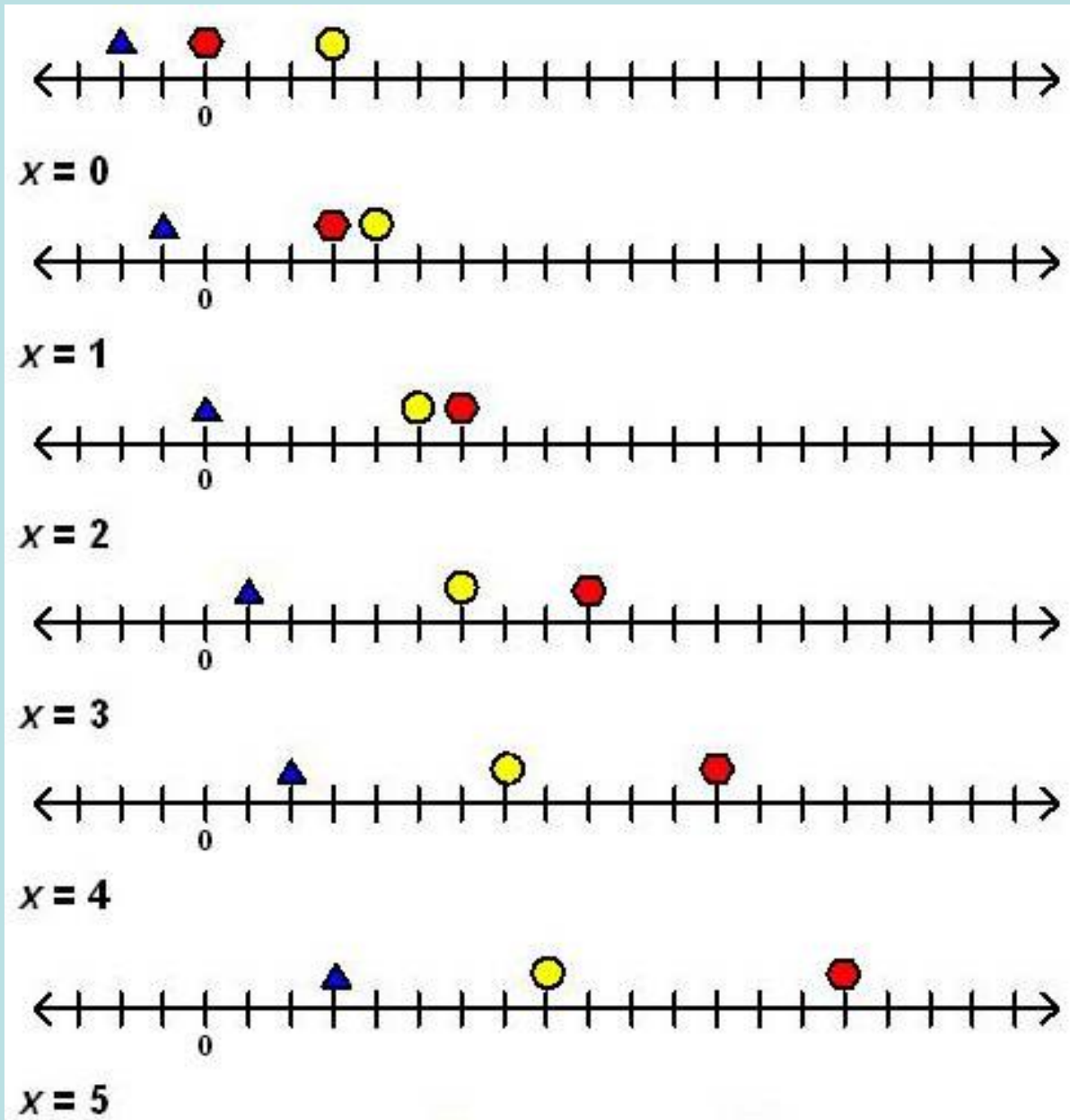
Comparing Growth Rates: x and $x+3$



Comparing Growth Rates: $x + 3$ and $3x$



Growth Rates: $x+3$, $3x$, and $x-2$



Growth Rates for $x+3$, $3x$, and $x-2$

Initial Student Conjectures

- $x + 3$ and $x - 2$ grow at the same rate as x .
- $3x$ grows at a much faster rate than $x + 3$ and $x - 2$.

Generalized Student Conjectures

x is a whole number

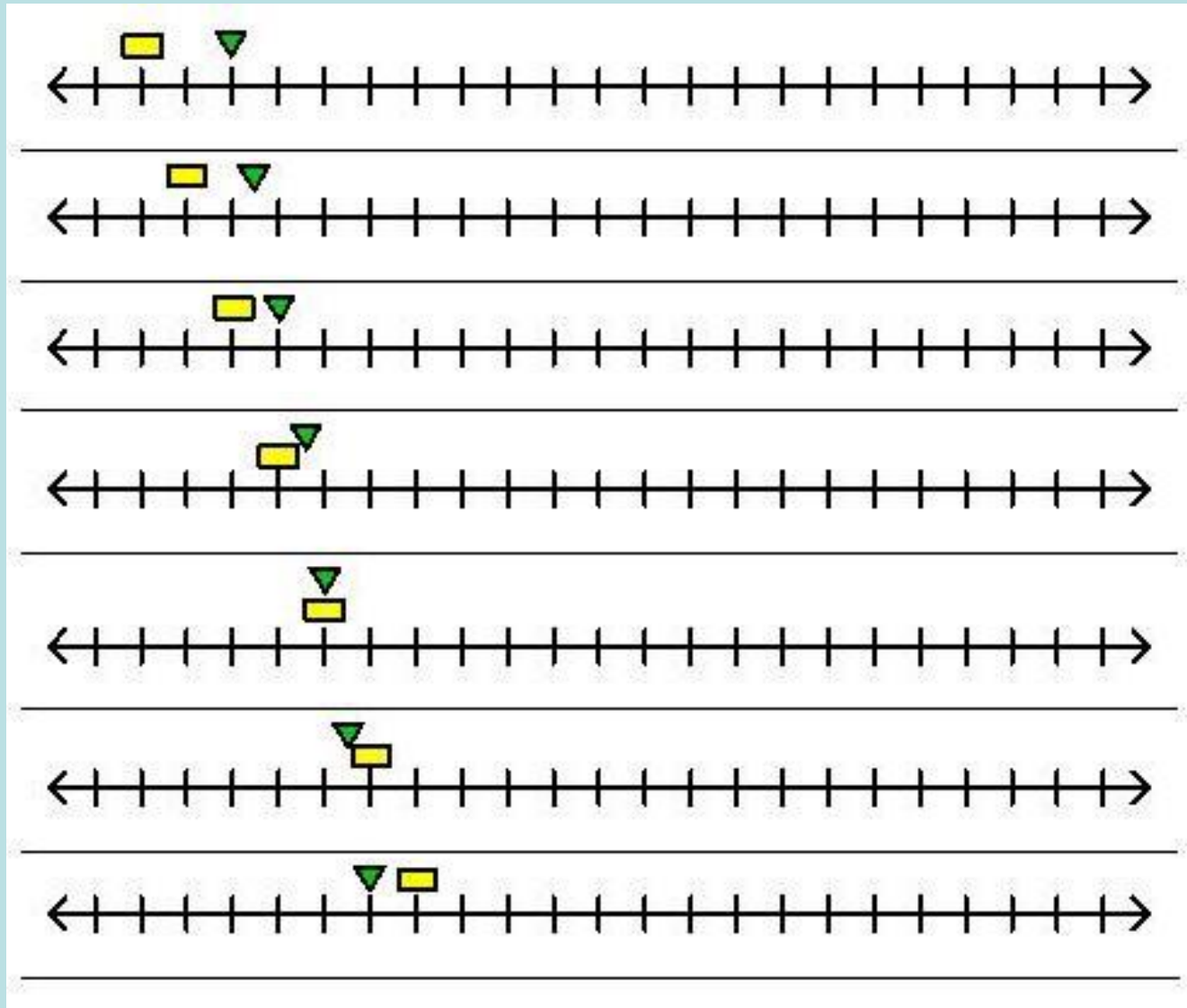
- The value of expressions with x plus or minus a positive number grow at the same rate as the graph of x .
- The value of expressions with x multiplied by a positive number start at 0 but pass and grow faster than the graph of x .
- When a positive number is added to a variable in an expression, the growth rate is slower than when the variable is multiplied by a positive number.

Benefits of Comparing Growth Rates of Variable Expressions

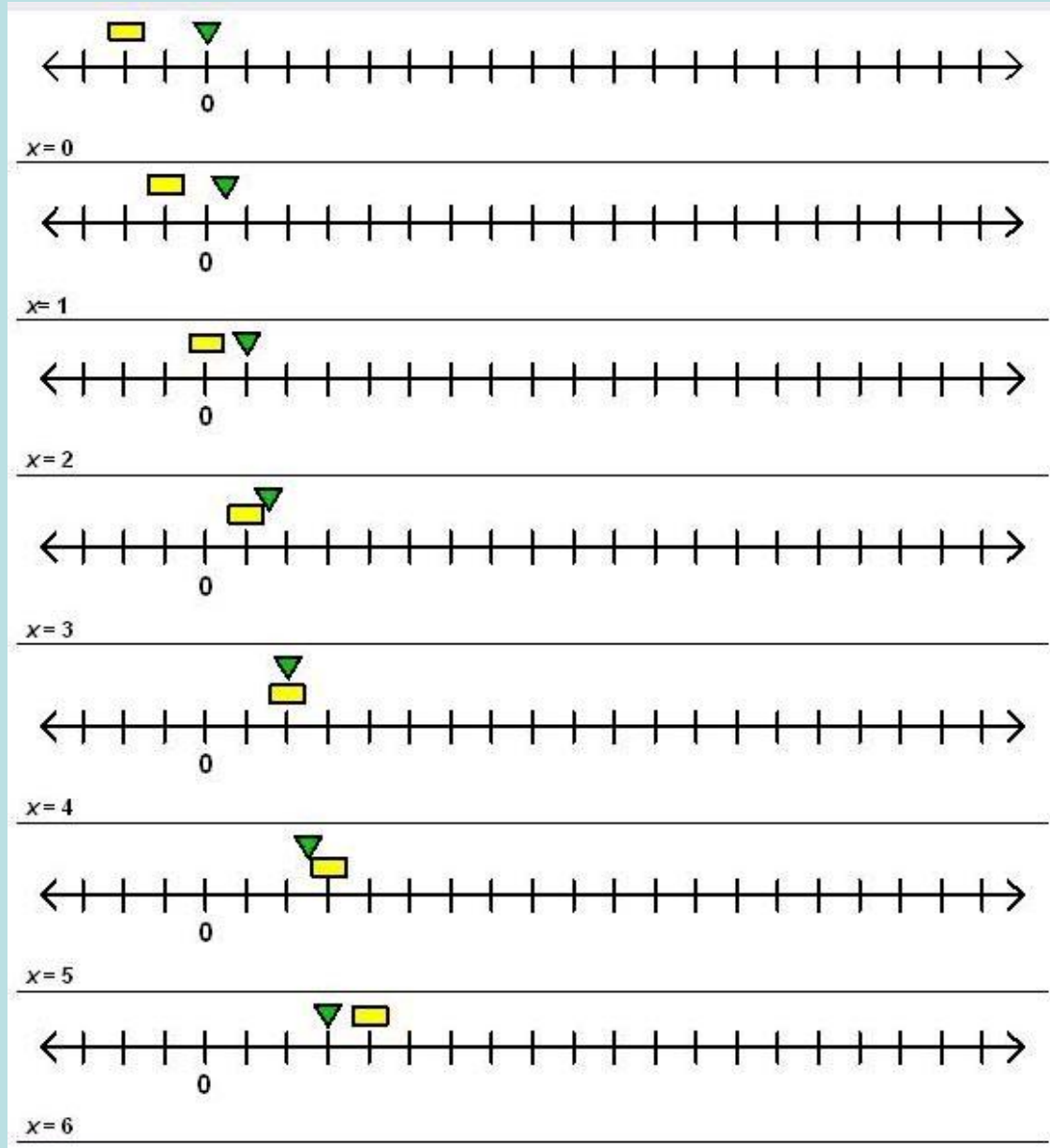
Students:

- evaluate and graph variable expressions.
- think more generally and more abstractly about rates of change/growth rates
- communicate their thinking: make mathematical conjectures, arguments, and conclusions.
- develop more comfort with and understanding of symbols (more symbol sense)

Comparing Growth Rates - Undo

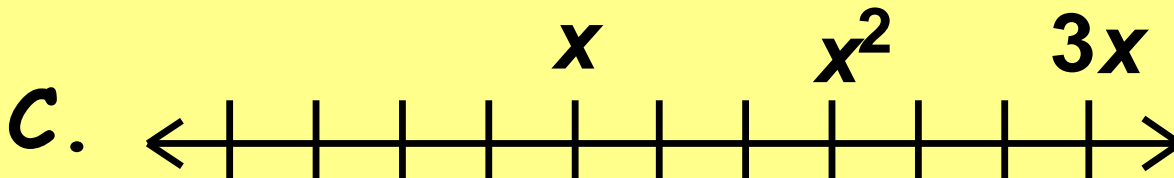
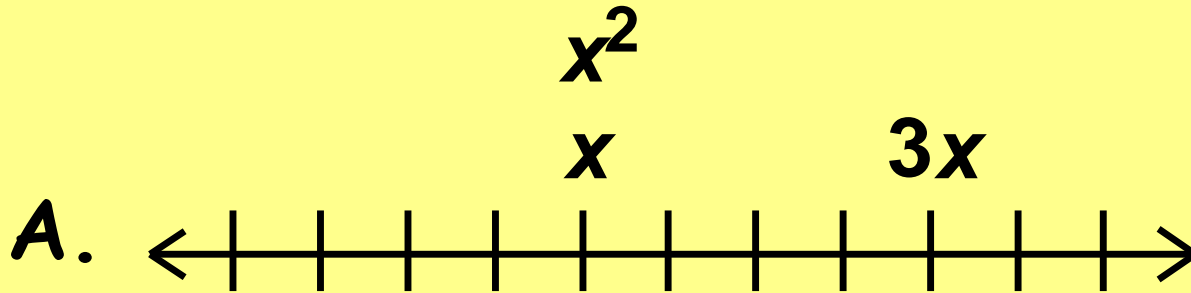


Comparing Growth Rates - Undo

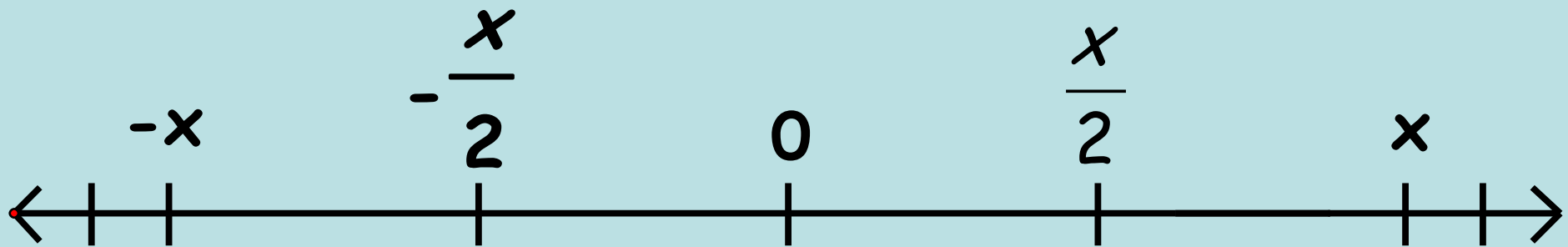
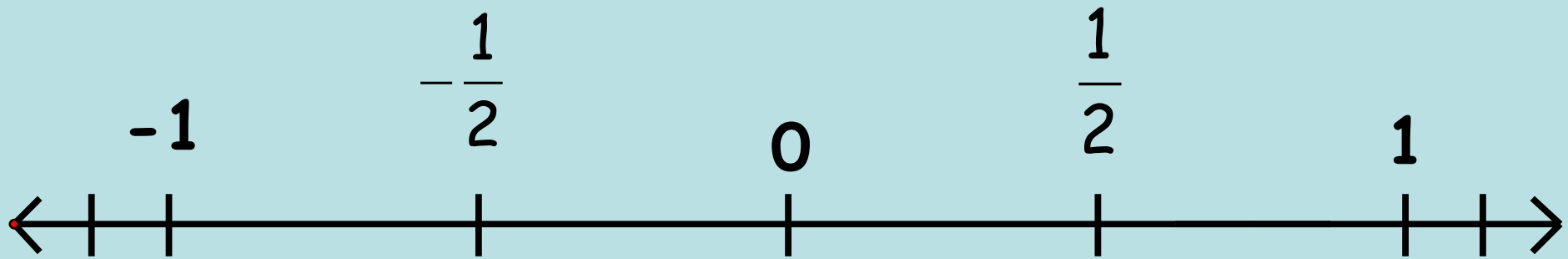


Variables on a Number Line

$x =$ a whole number

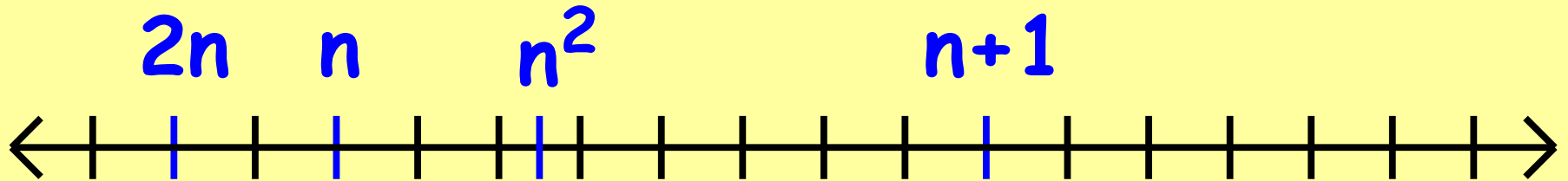
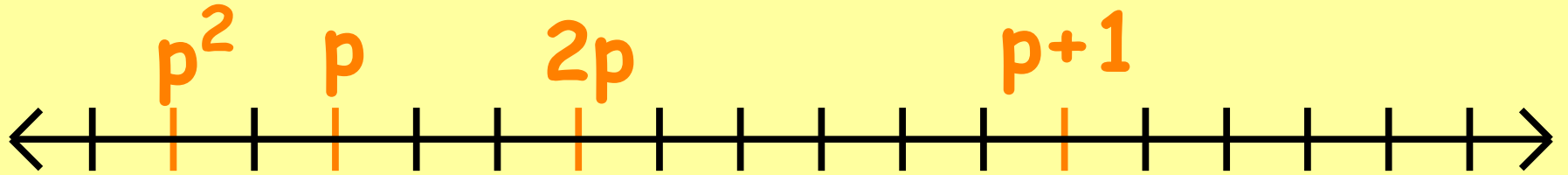


Variables on a Number Line



Variables on a Number Line

Note: p and n are rational numbers.

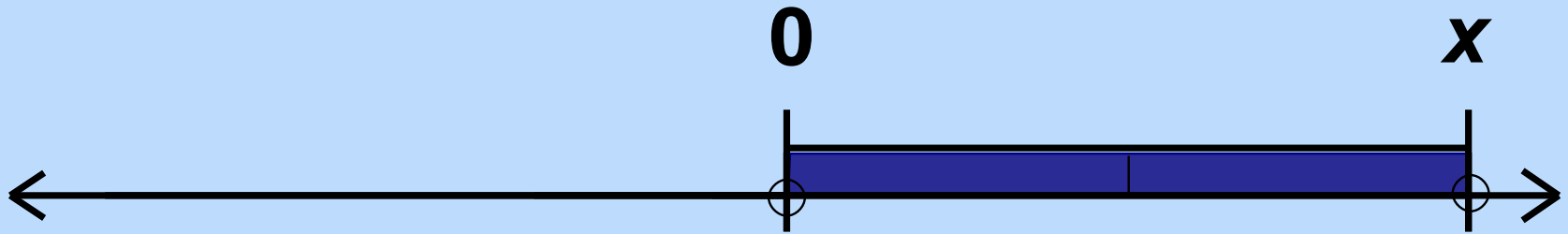


Benefits of Variables on a Scaled Number Line (1 value for variable)

Students

- apply what they know about numbers and operations in a given set and use scale to find the value of a variable.
- communicate their thinking: make mathematical conjectures, arguments, and conclusions.
- develop more comfort with and understanding of symbols (more **symbol sense**)

Variables on an Open Number Line



Where is $x - y$?

Variables on an Open Number Line with Integers



x and y are integers

Where is $x - y$?

How do you know?

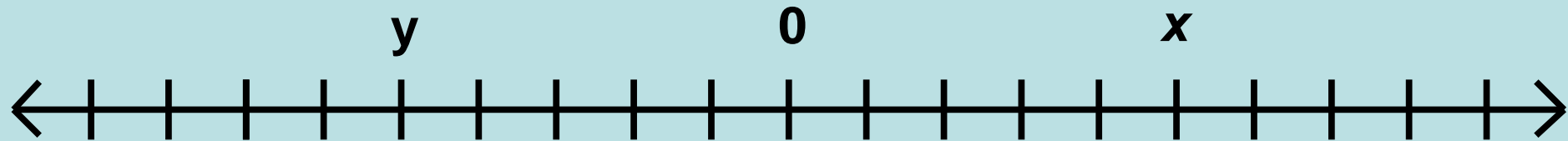
Benefits of Variables on an Open Number Line (more than 1 value for variable)

Students

- apply what they know to solve problems with a **set of answers**.
- communicate their thinking: make mathematical conjectures, arguments, and conclusions.
- develop more comfort with and understanding of symbols (**more symbol sense**)

Variables on a Number Line with Integers

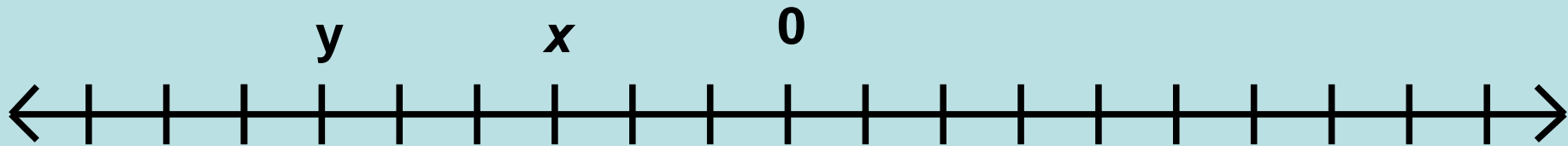
If x and y are integers,
what else do you know?



What is impossible to know?

Variables on a Number Line with Integers

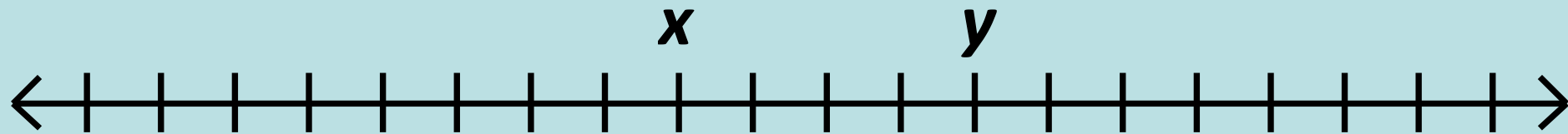
If x and y are integers, what
else do you know?



What is impossible to know?

Variables on a Number Line with Integers

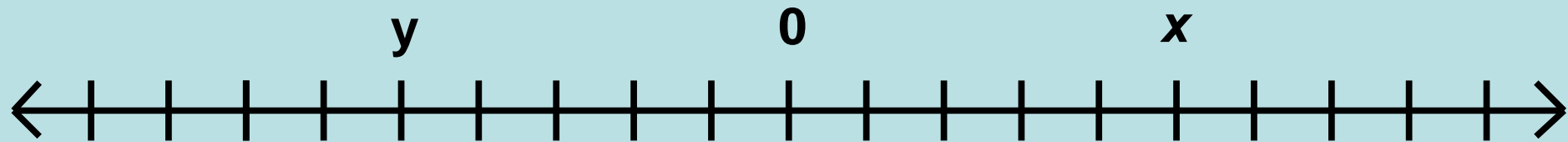
If x and y are integers, what
else do you know?



What is impossible to know?

Variables on a Number Line with Rational Numbers

If x is a rational number,
what else do you know?



What is impossible to know?

Benefits of working with Variable Expressions on a Scaled Number Line

- Working on a number line requires students to think more abstractly.
- Working on a number line requires students to consider what happens to numbers in a particular domain under different operations.
- Working with variable expressions on a number line helps develop **symbol sense**.

" Students bring two complementary abilities to bear on problems involving quantitative relationships: the ability to decontextualize — to abstract a given situation and represent it symbolically and manipulate the representing symbols as if they have a life of their own, without necessarily attending to their referents—and the ability to contextualize, to pause as needed during the manipulation process in order to probe into the referents for the symbols involved."

Parting Thoughts

Symbol sense is a comfort with symbols and symbol strings - coming from investigating their characteristics and using them in diverse situations.

It involves understanding different types of symbols and expressions, how they are related to each other, and how they can best be used to describe a particular situation.

Symbol sense is an attribute of all successful users of mathematics.

Sources

Boaler, Jo and Staples, Megan. "Creating Mathematical Futures through an Equitable Teaching Approach: The Case of Railside School", *Teachers College Record*, 2008.

CCSSM progression document: 6-8 Expressions and Equations at http://commoncoretools.files.wordpress.com/2011/04/ccss_progression_ee_2011_04_25.pdf

Chapin, S. and O'Connor, C. "Academically Productive Talk: Supporting Student Learning in Mathematics." in Martin, W.G., Strutchens, M., & Elliott, P. (Eds.), *The Learning of Mathematics*, Reston, VA: National Council of Teachers of Mathematics, 2007.

Sources(Cont'd)

- Hennessey, Charlie and Lawrence, Ann. *Lessons for Algebraic Thinking-Grades 6-8*. Sausalito, CA: Math Solutions Publications, 2002.
- Novak, Jodie and Jacobs, Judith. "Sorting Symbol Strings". *MTMS*, March 2005, 334-338.
- Schoenfeld, Alan and Abraham Arcavi. "The Meaning of Variable." *MT*. September 1988, 420-271.
- Williams, Jennifer Bay. "Developing Number Sense on the Number Line". *MTMS*. April 2001, 448-451.